Individual Finals B

1. Let a, b, c be real numbers in the interval $[0, 1]$, satisfying $ab + c \leq 1$. Find the maximal value of their sum $a + b + c$.

2. Let p be an odd prime. Prove that for every integer k, there exist integers a, b such that $p | a^2 + b^2 - k$.

3. Let ΔABC be a triangle, and let C_0, B_0 be the feet of perpendiculars from C and B onto AB and AC respectively. Let Γ be the circumcircle of ΔABC. Let E be a point on the Γ such that $AE \perp BC$. Let M be the midpoint of BC and let G be the second intersection of EM and Γ. Let T be a point on Γ such that TG is parallel to BC. Prove that T, A, B_0, C_0 are concyclic.