Algebra B

1. Let x, y be distinct positive real numbers satisfying
$$\frac{1}{\sqrt{x+y}-\sqrt{x-y}}+\frac{1}{\sqrt{x+y}+\sqrt{x-y}}=\frac{x}{\sqrt{y^3}}.$$
If $\frac{x}{y}=\frac{a+\sqrt{b}}{c}$ for positive integers a, b, c with $\gcd(a, c)=1$, find $a+b+c$.

2. Kris is asked to compute $\log_{10}(x^y)$, where y is a positive integer and x is a positive real number. However, they misread this as $(\log_{10} x)^y$, and compute this value. Despite the reading error, Kris still got the right answer. Given that $x > 10^{1.5}$, determine the largest possible value of y.

3. Compute the sum of all real numbers x which satisfy the following equation
$$\frac{8^x - 19\cdot 4^x}{16 - 25\cdot 2^x} = 2.$$

4. For a bijective function $g : \mathbb{R} \to \mathbb{R}$, we say that a function $f : \mathbb{R} \to \mathbb{R}$ is its *superinverse* if it satisfies the following identity $(f \circ g)(x) = g^{-1}(x)$, where g^{-1} is the inverse of g. Given $g(x) = x^3 + 9x^2 + 27x + 81$ and f is its superinverse, find $|f(-289)|$.

5. Let $f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4$ and let $\zeta = e^{2\pi i/5} = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$. Find the value of the following expression:
$$f(\zeta)f(\zeta^2)f(\zeta^3)f(\zeta^4).$$

6. The roots of a monic cubic polynomial p are positive real numbers forming a geometric sequence. Suppose that the sum of the roots is equal to 10. Under these conditions, the largest possible value of $|p(-1)|$ can be written as $\frac{m}{n}$, where m, n are relatively prime integers. Find $m+n$.

7. Consider the sum
$$S = \sum_{j=1}^{2021} |\sin\frac{2\pi j}{2021}|.$$
The value of S can be written as $\tan(\frac{c\pi}{d})$ for some relatively prime positive integers c, d, satisfying $2c < d$. Find the value of $c+d$.

8. Let f be a polynomial. We say that a complex number p is a *double attractor* if there exists a polynomial $h(x)$ so that $f(x) - f(p) = h(x)(x-p)^2$ for all $x \in \mathbb{R}$. Now, consider the polynomial
$$f(x) = 12x^5 - 15x^4 - 40x^3 + 540x^2 - 2160x + 1,$$
and suppose that it's double attractors are $a_1, a_2, \ldots, a_n$. If the sum $\sum_{i=1}^{n} |a_i|$ can be written as $\sqrt{a} + \sqrt{b}$, where a, b are positive integers, find $a+b$.