P U M ∴ C

Individual Finals B

1. Find all pairs of nonnegative integers (n, m) such that $2^n = 7^m + 9$.

2. Let $G = (V, E)$ be a simple connected graph. Show that there exists a subset of edges $F \subseteq E$ such that every vertex in $H = (V, F)$ has odd degree if and only if $|V|$ is even.

   Note: A *connected graph* is a graph such that any two vertices have a sequence of edges connecting one to the other.

   Note: A *simple graph* has no loops (edges of the form (v, v)) or duplicate edges.

3. Let MN be a chord of the circle Γ and let S be the midpoint of MN. Let A, B, C, D be points on Γ such that AC and BD intersect at S and A and B are on the same side of MN. Let d_A, d_B, d_C, d_D be the distances from MN to A, B, C, and D, respectively. Prove that $\frac{1}{d_A} + \frac{1}{d_D} = \frac{1}{d_B} + \frac{1}{d_C}$.