P U M ∴ C

Individual Finals A

1. Given the graph G and cycle C in it, we can perform the following operation: add another vertex v to the graph, connect it to all vertices in C and erase all the edges from C. Prove that we cannot perform the operation indefinitely on a given graph.

2. Prove that for every positive integer m, every prime p and every positive integer $j \leq p^{m-1}$, p^m divides $\binom{p^m}{pj} - \binom{p^{m-1}}{j}$.

3. Let $ABCDEF$ be a convex hexagon with area S such that $AB \parallel DE, BC \parallel EF, CD \parallel FA$ holds, and whose all angles are obtuse and opposite sides are not the same length. Prove that the following inequality holds: $A_{ABC} + A_{BCD} + A_{CDE} + A_{DEF} + A_{EFA} + A_{FAB} < S$, where A_{XYZ} is the area of triangle XYZ.